Role of charm in the decay of B mesons to $\eta' K$

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In this Brief Report we focus on the calculation of $f^{(c)}_{\eta'}$, the amplitude of the $(c\bar{c})\to$(gluons)$\to\eta'$ transition, which provides the magnitude of the contribution of the Cabibbo favored $b\to\bar{c}cs$ elementary process to $B\to\eta' K$ decay. It is found that $f^{(c)}_{\eta'}=-12.3$ to -18.4 MeV on the scale of m_c. This number is in strong contradiction with the estimations of Halperin and Zhitnitsky but almost in agreement with the phenomenological analysis of Feldman *et al.*, Petrov, and the estimations of Ali *et al.* [S0556-2821(99)03501-8]

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I. INTRODUCTION

Recently there has been a great theoretical interest [3–13] in the recently released experimental data on the branching ratios of the decays of $B\to K\eta'$ [1,2]:

$$B(B^{\pm}\to\eta' K^{\pm})=(6.5^{+1.5}_{-1.4}\pm 0.9)\times 10^{-5}, \qquad (1.1)$$

$$B(B^{0}\to\eta' K^{0})=(4.7^{+2.7}_{-2.0}\pm 0.9)\times 10^{-5}. \qquad (1.2)$$

In the standard model the Cabibbo favored $b\to\bar{c}cs$ elementary process may be followed by conversion of $\bar{c}c$ pair into η' through the gluons. The amplitude of this process is described by

$$M=\frac{G_F}{\sqrt{2}}V_{cb}V^*_{cs}a_1\langle\eta'(p)|\bar{c}\gamma_\mu\gamma_5 c|0\rangle \times\langle K(q)|\bar{s}\gamma_\mu b|B(p+q)\rangle. \qquad (1.3)$$

Here G_F is the weak coupling constant, V_{cb}, V^*_{cs} are Kobayashi-Maskawa matrix elements, $a_1=0.25$ phenomenological number obtained by a fit (see [3] for the references). The matrix element

$$\langle\eta'(p)|\bar{c}\gamma_\mu\gamma_5 c|0\rangle=-if^{(c)}_{\eta'}p_\mu \qquad (1.4)$$

is nonzero due to the virtual $\bar{c}c\to$gluons transitions. Certainly, this matrix element is suppressed by the $1/m_c^2$ factor. However, because of strong nonperturbative gluon fields together with the Cabibbo favored $b\to c$ transition the suggested mechanism (1.3) can be expected to compete appreciably with the other mechanisms of the $B\to K\eta'$ process [9,12].

If we assume the dominance of the mechanism (1.3) the branching ratio is written in terms of $f^{(c)}_{\eta'}$ as [3]

$$Br(B\to K\eta')\simeq 3.92\times 10^{-3}\left(\frac{f^{(c)}_{\eta'}}{1\ \text{GeV}}\right)^2. \qquad (1.5)$$

Using the data (1.1) it is found $f^{(c)}_{\eta'}\simeq 140$ MeV (''expt'').

This value perfectly coincides with estimation of Halperin and Zhitnitsky [3]:

$$f^{(c)}_{\eta'}=(50\text{–}180)\ \ \text{MeV}. \qquad (1.6)$$

On the other hand, a recent phenomenological study placed a bound on $f^{(c)}_{\eta'}$, namely $-65\ \text{MeV}\leqslant f^{(c)}_{\eta'}\leqslant 15$ MeV, with $f^{(c)}_{\eta'}$ being consistent with zero by analyzing the Q^2 evolution of the $\eta'\gamma$ form factor [7], and more recently it was estimated from observed ratio of J/ψ decay to η' and η_c the value of $f^{(c)}_{\eta'}=-(6.3\pm 0.6)$ MeV [8]. Other similar estimation which leads to $|f^{(c)}_{\eta'}|<12$ MeV was made in [13]. Ali *et al.* considered the complete amplitude for the exclusive B-meson decays, including $\eta' K$ channels, where it was combined the contribution from the process $b\to s(c\bar{c})\to s(gluons)\to s\eta^{(\prime)}$ with all the others arising from the four-quark and chromomagnetic operators, as detailed in their papers [5,6]. Their estimations gave $|f^{(c)}_{\eta'}|\simeq 5.8$ MeV [5] and $f^{(c)}_{\eta'}=-3.1(-2.3)$ MeV (for m_c in the range 1.3–1.5 GeV) [6] in agreement with the analysis [7]. They stressed the importance of the sign of $f^{(c)}_{\eta'}$ and found a theoretical branching ratio in the range

$$B(B\to\eta' K)=(2-4)\times 10^{-5},$$

which is somewhat smaller than the experimental one (1.1). The similar analysis made in [10] leads the authors to conclude that $f^{(c)}_{\eta'}=-50$ MeV may provide the explanation of the data.

Having this situation, it is important to recalculate $f^{(c)}_{\eta'}$ [1] to clarify the mechanism of $B\to K\eta'$ decay in the similar framework performed by Halperin and Zhitnitsky [3].

II. CALCULATIONS OF $f^{(c)}_{\eta'}$

The symmetry of the classical Lagrangian may be destroyed by quantum fluctuations [15–17]. In gauge theories the axial anomaly arises from noninvariance of the fermionic

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[1] The details of the present calculations are given in [14].

measure against axial transformations in the path integrals of the theory [18] (see also Ref. [19], concerning higher-loop corrections). The present problem is intimately related with this phenomena.

In the following we will work only with the Euclidian QCD.[2] In the Euclidean QCD the axial anomaly in the light quark axial current in chiral limit reads

$$\partial_\mu \psi_f^\dagger \gamma_5 \gamma_\mu \psi_f = -i\frac{g^2}{16\pi^2} G\tilde{G}, \tag{2.1}$$

where ψ_f is the light quark field ($f=u,d,s$) and g the QCD coupling constant. $2G\tilde{G}=\epsilon^{\mu\nu\lambda\sigma}G^a_{\mu\nu}G^a_{\lambda\sigma}$, where $G^a_{\mu\nu}$ is the gluon field strength operator with a being the color index.

The situation with heavy quarks is very different, since we must take into account the contribution of the mass term. The divergence of the axial current of charmed quarks has a form:

$$\partial_\mu c^\dagger \gamma_\mu \gamma_5 c = -i\frac{g^2}{16\pi^2} G\tilde{G} + 2m_c c^\dagger \gamma_5 c, \tag{2.2}$$

The first term in Eq. (2.2) again comes from noninvariance of the fermionic measure (or in other words, from Pauli-Villars regularization). The main problem here is to calculate the contribution from the second term in Eq. (2.2). It is clear that this one is reduced to the problem of the calculation of the vacuum expectation value of the operator $2m_c c^\dagger \gamma_5 c$ in the presence of gluon fields.

In the path integral approach the calculation of the contribution of this term to any matrix element over light hadrons may be considered in sequence of the integrations. First the integration over c-quark is performed, and the next step is the calculation of the integral over gauge gluon field and finally integral over light quarks.

We consider here the first step: the integration over c quarks. We define

$$\langle 2m_c c^\dagger(x)\gamma_5 c(x)\rangle = \int DcDc^\dagger 2m_c c^\dagger(x)\gamma_5 c(x) \times \exp\left(\int c^\dagger (i\hat{\nabla} + im_c)c\right). \tag{2.3}$$

Here $i\hat{\nabla}=\gamma_\mu(i\partial_\mu + gA_\mu)$ and we introduce the operator P_μ and p_μ which are defined in the coordinate space as $\langle x|P_\mu|y\rangle = i\nabla_\mu \delta(x-y)$ and $\langle x|p_\mu|y\rangle = i\partial_\mu\delta(x-y)$. It is clear that the formal answer for the path integral (2.3) can be written in the form

$$\langle 2m_c c^\dagger(x)\gamma_5 c(x)\rangle = 2m_c \det\|\hat{P}+im_c\| \times \langle x|\mathrm{Tr}\ \gamma_5(\hat{P}+im_c)^{-1}|x\rangle \tag{2.4}$$

[2]We are using here the convention of the Euclidian QCD: $ix_{M0}=x_{E4}$, $x_{Mi}=x_{Ei}$, $A_{M0}=iA_{E4}$, $A_{Mi}=-A_{Ei}$, $\psi_M=\psi_E$, $i\overline{\psi}_M=\psi_E^\dagger$, $\gamma_{M0}=\gamma_{E4}$ $\gamma_{Mi}=i\gamma_{Ei}$, $\gamma_{M5}=\gamma_{E5}$. We will omit index E.

$\det\|\hat{P}+im_c\|$ must be regularized in the standard manner as

$$\det\|\hat{P}+im_c\| \to \det\left\|\frac{(\hat{P}+im_c)(\hat{p}+iM)}{(\hat{p}+im_c)(\hat{P}+iM)}\right\|,$$

where M is the regulator mass. Equation (2.4) must be a gauge invariant function of the gauge field A and therefore must be expressed through the gluon field strength tensor and their covariant derivatives.

We will follow the operator method proposed by Vainshtein *et al.* [20] in the same line as in [3]. The key ingredient of this method is based on an assumption of a possibility of an expansion of Eq. (2.4) over gG/m_c^2. We will take into account $O(g^2G^2)$ and $O(g^3G^3)$ terms in the calculations of Eq. (2.4). We start from the calculation of

$$H(x) = 2m_c\langle x|\mathrm{Tr}\ \gamma_5(\hat{P}+im_c)^{-1}|x\rangle \tag{2.5}$$

$$= -2im_c^2\langle x|\mathrm{Tr}\ \gamma_5\left(P^2+m_c^2+\frac{1}{2}\sigma gG\right)^{-1}|x\rangle$$

$$= H_2(x)+H_3(x)+O(g^4G^4), \tag{2.6}$$

where

$$H_2(x) = -2im_c^2\langle x|\mathrm{Tr}\ \gamma_5(P^2+m_c^2)^{-2} \times\frac{g}{2}\sigma G(P^2+m_c^2)^{-1}\frac{g}{2}\sigma G|x\rangle \tag{2.7}$$

and

$$H_3(x) = 2im_c^2\langle x|\mathrm{Tr}\ \gamma_5(P^2+m_c^2)^{-2} \times\frac{g}{2}\sigma G(P^2+m_c^2)^{-1}\frac{g}{2}\sigma G(P^2+m_c^2)^{-1} \times\frac{g}{2}\sigma G|x\rangle. \tag{2.8}$$

Here $\sigma G=\sigma_{\mu\nu}G_{\mu\nu}, \sigma_{\mu\nu}=(i/2)[\gamma_\mu,\gamma_\nu]$.

It is straightforward to calculate $H_3(x)$, since we may neglect the noncommutativity of the operators in Eq. (2.8) and replace P operator by p in Eq. (2.8). In that case we may use the evident formulas,

$$\langle x|\frac{1}{(p^2+m^2)^n}|x\rangle = \int\frac{d^4p}{(2\pi)^4}\frac{1}{(p^2+m^2)^n} = (2^4\pi^2(n-1)(n-2)m^{2(n-2)})^{-1},$$

$$\mathrm{Tr}\ \gamma_5(\sigma G)^3 = i2^5\mathrm{tr}_c G\tilde{G}G,$$

where $G\tilde{G}G=G_{\mu\nu}\tilde{G}_{\nu\rho}G_{\rho\mu}$. We get

$$H_3(x) = -i\frac{g^3}{2^4 3\pi^2 m_c^2} f_{abc}G^a\tilde{G}^bG^c. \tag{2.9}$$

The calculation of $H_2(x)$ needs much more efforts. First of all we represent the $\sigma G(P^2+m_c^2)^{-1}$ in the form

$$\sigma G(P^2+m_c^2)^{-1}=(P^2+m_c^2)^{-1}\sigma G+(P^2+m_c^2)^{-2}[P^2,\sigma G]+(P^2+m_c^2)^{-3}[P^2,[P^2,\sigma G]]+\dots. \tag{2.10}$$

The routine calculations of the commutators in Eq. (2.10) lead to

$$[P^2,\sigma G]=\nabla^2\sigma G+2iP_\alpha\nabla_\alpha\sigma G, \tag{2.11}$$

$$[P^2,[P^2,\sigma G]]=\nabla^4\sigma G+2iP_\alpha(\nabla_\alpha\nabla^2+\nabla^2\nabla_\alpha)\sigma G+(2i)^2P_\beta G_{\beta\alpha}\nabla_\alpha\sigma G+(2i)^2P_\alpha P_\beta\nabla_\beta\nabla_\alpha\sigma G. \tag{2.12}$$

Other higher commutators lead to the terms order $O(G^3)$ in the expansion (2.10) and may be neglected. Following the arguments of [20] we may neglect also the terms which contains single operator P_μ. The reason is that the matrix elements

$$\langle x|(P^2+m_c^2)^{-n}P_\mu|x\rangle\sim\nabla_\mu G^2.$$

By using the Bianchi identity it is easy to show that

$$\nabla^2 G_{\mu\nu}=i(G_{\alpha\nu}G_{\alpha\mu}+G_{\alpha\mu}G_{\nu\alpha})-\nabla_\nu\nabla_\alpha G_{\alpha\mu}-\nabla_\mu\nabla_\alpha G_{\nu\alpha}. \tag{2.13}$$

It is evident that the term $\nabla^4\sigma G$ is order of $O(G^3)$ and may be neglected. Collecting all of the $O(g^2G^2)$ and $O(g^3G^3)$ terms in $H_2(x)$ in Eq. (2.7), we get

$$H_2(x)=\frac{ig^2}{2^4\pi^2}G^a\tilde{G}^a+\frac{ig^3}{2^5 3\pi^2 m_c^2}f_{abc}G^a\tilde{G}^bG^c. \tag{2.14}$$

Finally

$$H(x)=\frac{ig^2}{2^4\pi^2}G^a\tilde{G}^a-\frac{ig^3}{2^5 3\pi^2 m_c^2}f_{abc}G^a\tilde{G}^bG^c. \tag{2.15}$$

We neglect here the small contributions of the terms like $\nabla_\mu\nabla_\alpha G_{\nu\alpha}$.

As expected, the first term in $H(x)$ cancels with the first term in Eq. (2.2), which is the contribution from noninvariance of the measure and the rest part leads to the divergence of the c-quark axial current in the form

$$\langle\partial_\mu c^\dagger(x)\gamma_\mu\gamma_5 c(x)\rangle=-\frac{ig^3}{2^5 3\pi^2 m_c^2}f_{abc}G^a\tilde{G}^bG^c. \tag{2.16}$$

We would like to stress an attention that our answer for $\langle\partial_\mu c^\dagger(x)\gamma_\mu\gamma_5 c(x)\rangle$ is 6 times less than was calculated by Halperin and Zhitnitsky [3].

We apply this result to the calculation of the $f_{\eta'}^{(c)}$ and compare with the analogous quantity $f_{\eta'}^{(u)}$, which is defined in the similar way as $f_{\eta'}^{(c)}$ in Eq. (1.4). These quantities certainly are defined in Minkowski space. Equation (2.1) and (2.16) may be easily translated from Euclidian to Minkowski space accordingly the above-given prescription. For instance $[(G^a\tilde{G}^a)_E\to(G^a\tilde{G}^a)_M]$ and $[(f_{abc}G^a\tilde{G}^bG^c)_E\to-(f_{abc}G^a\tilde{G}^bG^c)_M]$, which lead to

$$m_{\eta'}^2 f_{\eta'}^{(u)}=\langle 0|\frac{g^2}{16\pi^2}(G^a\tilde{G}^a)_M|\eta'\rangle \tag{2.17}$$

and

$$m_{\eta'}^2 f_{\eta'}^{(c)}=\langle 0|\frac{g^3}{2^5 3\pi^2 m_c^2}(f_{abc}G^a\tilde{G}^bG^c)_M|\eta'\rangle. \tag{2.18}$$

The phenomenological way of the estimation of the $f_{\eta'}^{(u)}$ is the application of the QCD+QED axial anomaly equation together with data on $\eta'\to 2\gamma$ decay leads to

$$f_{\eta'}^{(u)}=63.6\ \text{MeV}, \tag{2.19}$$

which was used in [6]. On the other hand the calculation of the matrix elements (2.17), (2.18) may be reduced to the calculation of the correlators

$$\langle G^a\tilde{G}^a(x)G^a\tilde{G}^a(y)\rangle,\quad f_{abc}G^a\tilde{G}^bG^c(x)G^a\tilde{G}^a(y)\rangle$$

respectively. The calculation of the correlators are naturally performed in Euclidian space. These correlators have almost the same dependence on the large relative distances $|x-y|$ and the ratio of these correlators become almost constant, at least in DP chiral quark model [22,23]. So, in this model, which was successfully checked by the calculation of the axial anomaly low-energy theorem [21] in chiral limit, the ratio of Eqs. (2.18) and (2.17) is equal to

$$\frac{f_{\eta'}^{(c)}}{f_{\eta'}^{(u)}}=-\frac{12}{5\rho^2}\frac{1}{6m_c^2}\sim-0.1. \tag{2.20}$$

Here ρ is the average size of the QCD vacuum instantons, for which phenomenological analysis, variational and lattice calculations showed that

$$\rho=1/3\ \ \text{fm}, \tag{2.21}$$

and we used this value for the estimation (2.20). So, By taking into account the estimation (2.19) [we use $m_c(\mu_1\simeq m_c)\simeq 1.25$ GeV on the scale $\mu_1\simeq m_c$ for the numerical estimates], we find

$$f_{\eta'}^{(c)}=-6\ \ \text{MeV}. \tag{2.22}$$

This number is close to the one of [5], $|f_{\eta'}^{(c)}|=5.8$ MeV and the sign and the order of the value coincide with the estimations of [6,8,13].

Recently, Shuryak and Zhitnitsky [4] performed direct numerical evaluations of the various correlators of the operators $g^2G^a\tilde{G}^a$, $g^3f_{abc}G^a\tilde{G}^bG^c$ in the Interacting Instanton Liquid Model (IILM). Their calculations lead to

$$\langle 0|g^2G^a\tilde{G}^a|\eta'\rangle=7\ \ \text{GeV}^3 \tag{2.23}$$

(which leads to $f_{\eta'}^{(u)} = 48.3$ MeV) and

$$\frac{|\langle 0|g^3 f_{abc} G^a \tilde{G}^b G^c|\eta'\rangle|}{|\langle 0|g^2 G^a \tilde{G}^a|\eta'\rangle|} \approx (1.5\text{–}2.2)\ \ \mathrm{GeV}^2. \qquad (2.24)$$

The later is somewhat large than their simple estimate for this ratio of matrix elements

$$\frac{12}{5}\left\langle \frac{1}{\rho^2}\right\rangle \approx (1 \sim 1.5)\ \ \mathrm{GeV}^2. \qquad (2.25)$$

With the use of Eq. (2.24) [instead of the factor $12/5\rho^2$ in Eq. (2.20)] we arrive at

$$\frac{f_{\eta'}^{(c)}}{f_{\eta'}^{(u)}} = -0.17 \ \text{ to } \ -0.25. \qquad (2.26)$$

This ratio gives $f_{\eta'}^{(c)} = -8.2 \sim -12.3$ MeV at the scale of the size of the instanton $\mu_2 \approx \rho^{-1}$. The abovementioned experimental numbers (1.1) are given at the scale $\mu_1 \approx m_c$, which is different from the scale of this instanton calculation. The account of the anomalous dimension of the $g^3 G\tilde{G}G$ operator leads to correction [4]

$$f_{\eta'}^{(c)}(\mu_1 \simeq m_c) \simeq 1.5 f_{\eta'}^{(c)}(\mu_2 \simeq \rho^{-1}). \qquad (2.27)$$

The account of this scale factor leads to

$$f_{\eta'}^{(c)}(\mu_1 \simeq m_c) = -12.3 \ \text{ to } \ -18.4 \ \ \mathrm{MeV}. \qquad (2.28)$$

Hence, using Eq. (2.24), the result of more sophisticated calculations of Shuryak and Zhitnitsky, we get the number (2.28) which is 2–3 times larger than simple estimation (2.22).

These numbers (2.22), (2.28) are in agreement with the phenomenological bounds [7,13] and almost in agreement in the sign and the value with [6,8] but six–ten times less than the estimations given by [3] (see also [4]).

By using the numerical analysis of the branching ratio for $B^{\pm} \to \eta' K^{\pm}$ given at [5] (Fig. 17 of [5]) we expect that the value of $f_{\eta'}^{(c)}$ given in Eq. (2.28) may provide a more satisfactory explanation of the experimental data.[3] We reserve this investigation for the future publication.

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[3] Quite recently Cheng and Tseng [11] concluded an impressively good explanation of these data using our value (2.28) for $f_{\eta'}^{(c)}$.

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